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Computer Interface in Euler and Fourth-order Runge - Kutta Method

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Abstract: Differential equations are commonly used for mathematical modelling in science and engineering. In most real life situations, the differential equation that models the problem is complex to solve exactly, only a limited number of differential equations can be solved analytically. Large number of ordinary differential equations whose solutions cannot be obtained by regular analytical methods where we have to use the numerical methods to get the approximate solution of a differential equation under the prescribed initial conditions. This paper mainly presents Euler's method and fourth-order Runge -Kutta Method (RK4) for solving initial value problems (IVP) for ordinary differential equations (ODE). In order to verify the accuracy, compare numerical solutions with the exact solutions. This paper will then proceed to explain what steps the methods actually carries out in solving the differential equation along with the high level programming language code. The programme result is in good agreement with the numerical solutions and exact solutions.

Keywords: Initial value problem, Euler's Method, Runge - Kutta Method.

I. INTRODUCTION

An equation that consists of derivatives is called a differential equation. Differential equations have applications in all areas of science and engineering. Mathematical formulation of most of the physical and engineering problems lead to differential equations. So, it is important for engineers and scientists to know how to set up differential equations and solve them. Differential equations are of two types

1) Ordinary differential equation (ODE)
2) Partial differential equations (PDE)

Numerical methods for ordinary differential equations are methods used to find numerical approximations to the solutions of ordinary differential equations (ODEs). Many differential equations cannot be solved using symbolic computation (analysis). For practical purposes, however – such as in engineering – a numeric approximation to the solution is often sufficient. Many authors have attempted to solve initial value problems (IVP) to obtain high accuracy rapidly by using numerical methods, such as Euler method and Runge - Kutta method, and also some other methods. In this paper Euler method and Runge - Kutta method are applied for solving ordinary differential equations in initial value problems. Based on the Programming Language to solve them to get exact value and approximated values by Euler and Runge Kutta fourth order method, also studied whether the accuracy obtained by Runge Kutta fourth order method can be achieved by increasing the number of intervals[1][2].

A. Euler's Method

This process is very slow and to obtain accuracy the value of 'h' must be very small because of this restriction on 'h' the method is unsuitable for practical use.

Formula $y_{(n+1)}(x)= y_n +h\, f(x_n, y_n)$.

Usually, the above equation is the better one by far, we discuss several algorithms that are vastly superior to the Euler method [3][4][5][6].

In the meantime, it is easy to write a computer program to carry one the calculations required to produce the results in Tables (c) (d) (e) & (f)

The algorithm of Euler method is given below

STEP 1:**define** f(t, y)
STEP 2 : **input** initial value t0 & y0
STEP 3: **input** step size h and number of steps n
STEP 4: **output** t0&y0
STEP 5: for j from 1 to n do
STEP 6: k1 = f(t, y)
y = y + h*k1

t = t + h
STEP 7: **output** t & y
STEP 8: **end**

B. **Runge-Kutta Method**

As already mentioned, Euler's method is less efficient in practical problems since it requires h to be small. The Runge Kutta method is most popular because it is quite accurate, stable and easy to program. The fourth order Runge Kutta method (RK4) is widely used for solving initial value problems (IVP) for ordinary differential equation (ODE).
The general formula for Runge Kutta approximation is

$$y_{(n+1)}(x)=y_n+(k_1+2k_2+2k_3+k_4)$$

Where $k_1=hf(x_0,y_0)$

$k_2=h\ f(x_{0\ +},\ y_{0\ +}k_1)$

$k_3=\ h\ f(x_{0\ +},\ y_{0\ +}k_2)$

$k_4=hf(x_0+h,y_0+k_3)$

Where k is the weighted mean of $k_1.k_2,k_3,$and k_4.
Hence $y_{n+1}=y_n+k$.
Step 6 in Euler algorithm must be replaced as follows:
STEP 1: **define** f(t,y)
STEP 2 : **input** initial value t0 & y0
STEP 3: **input** step size h and number of steps n
STEP 4: **output** t0&y0
STEP 5: for j from 1 to n do
STEP 6: k1 = f(t,y)
k2 = f(t+0.5*h,y+0.5*h*k1)
k3 = f(t+0.5*h,y+0.5*h*k2)
k4 =f(t+h,y+h*k3)
y=y+(h\6)*(k1+2*k2+2*k3+k4)
t = t+h
STEP 7: **output** t & y
STEP 8: **end**
EXAMPLE
Result for the numerical solution, exact solution & program output of $y^{I} = x + y$; $y(0) = 1$
The exact solutions of the given problem is $y(x) = 2e^x - (x+1)$
Using the Euler method & Fourth order Runge - Kutta Method for step sizes h =0.1 & h =0.05.

The approximate results are obtained as shown in Tables I & II.

TABLE I
Numerical approximations for step size *h* = 0.1[6].

X_n	Euler Method $y(x_n)$	R.K.method $Y(x_n)$	Exact Solution y_n
0.1	1.1	1.1103	1.1103
0.2	1.22	1.3021	1.2428
0.3	1.362	1.544	1.3997
0.4	1.5282	1.7434	1.5836

TABLE II
Numerical approximations for step size *h*= 0.05[6].

X_n	Euler Method $y(x_n)$	R.K.method $Y(x_n)$	Exact Solution y_n
0.05	1.105	1.0525	1.0525
0.1	1.3152	1.1078	1.1103
0.15	1.2309	1.1709	1.1736
0.2	1.3024	1.2373	1.2428

The following values are obtained based on programme results for Euler's method as show in Figures 1,2,3,4.

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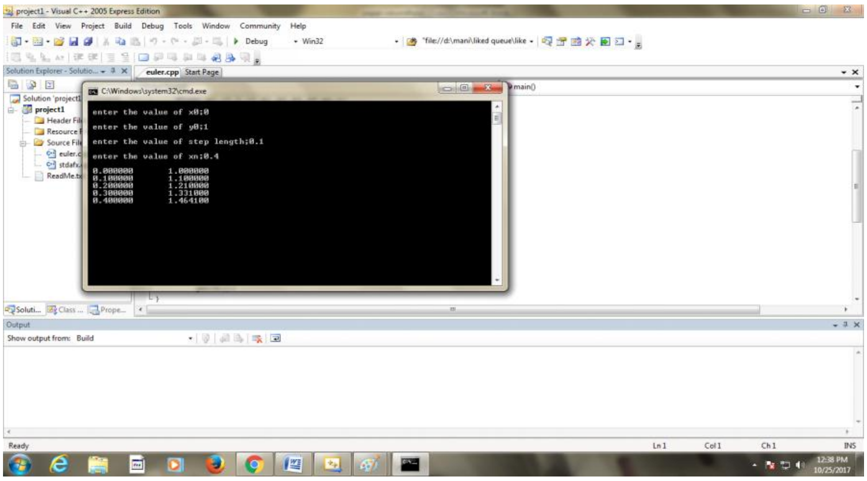

Fig.1 Program results for step size h =0.1($x_{n=0.1}$)

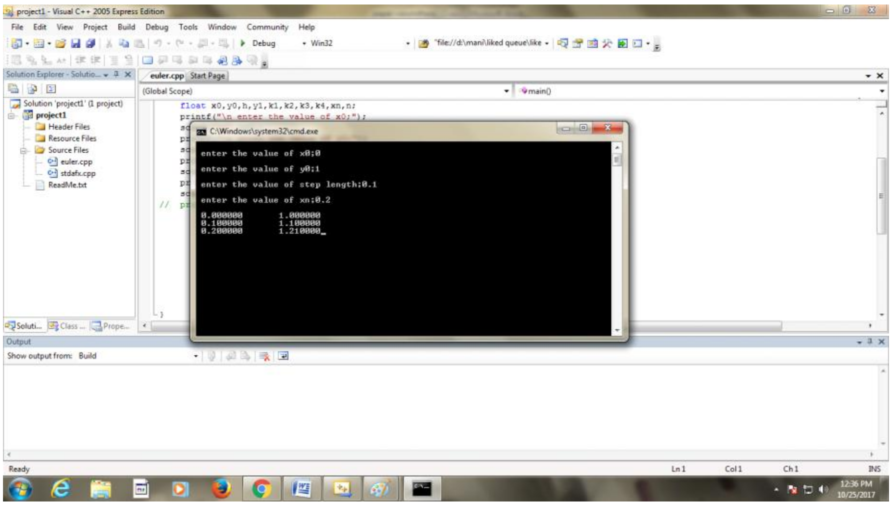

Fig.2 Program results for step size h =0.1($x_{n=0.2}$)

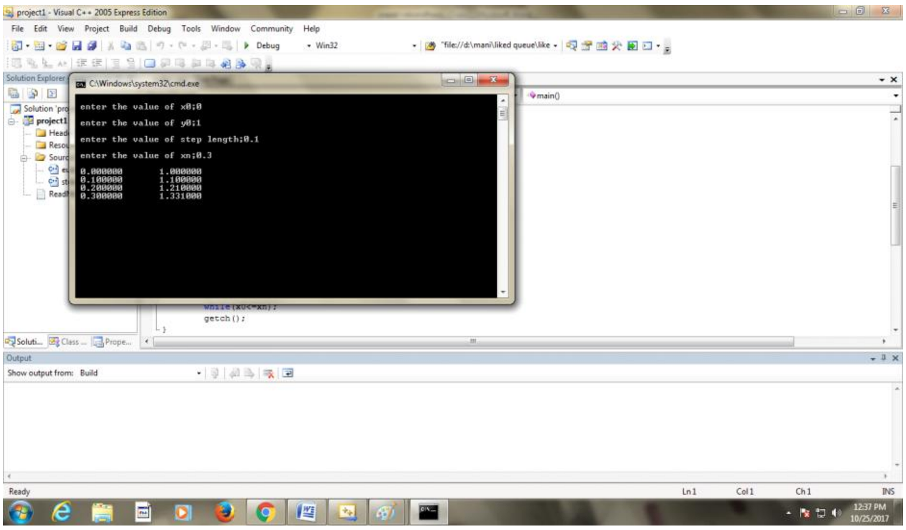

Fig.3 Program results for step size h =0.1($x_{n=0.3}$)

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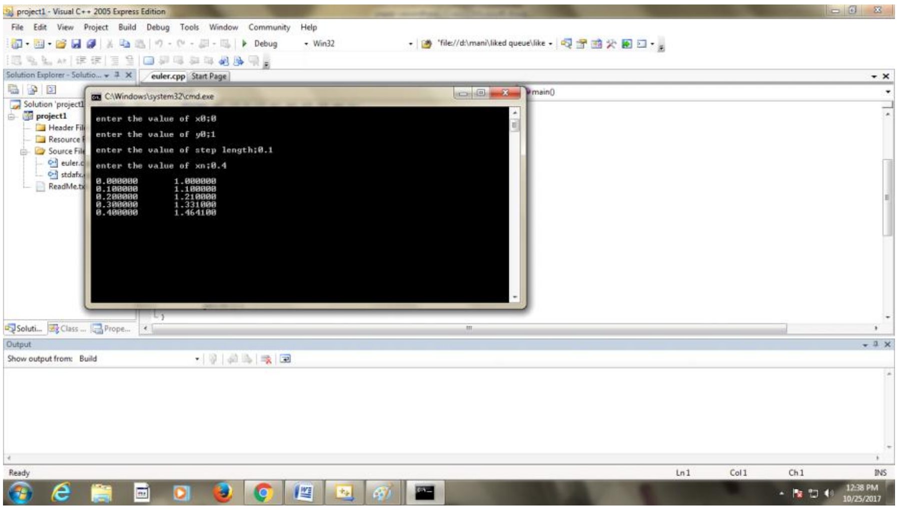

Fig.4 Program results for step size h =0.1($x_{n = 0.4}$)

The following values are obtained based on program results for RK method as show in Figures 5 & 6

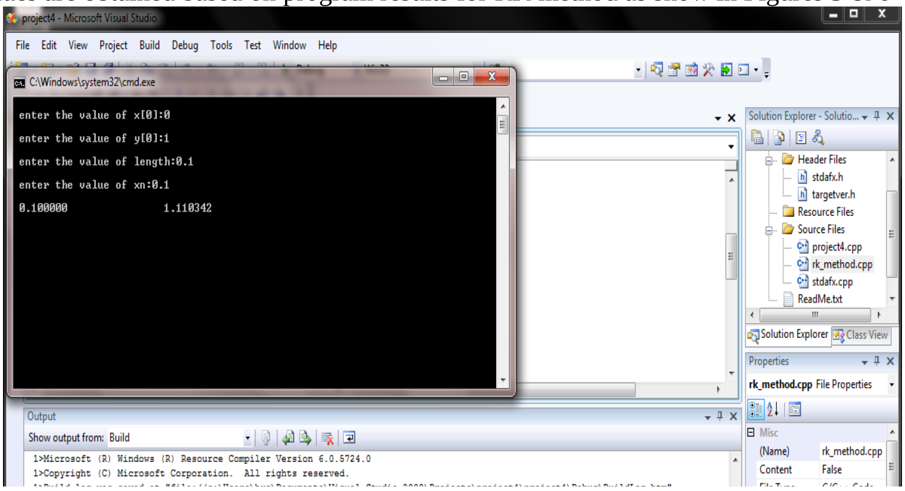

Fig.5 Program results for step size h =0.1

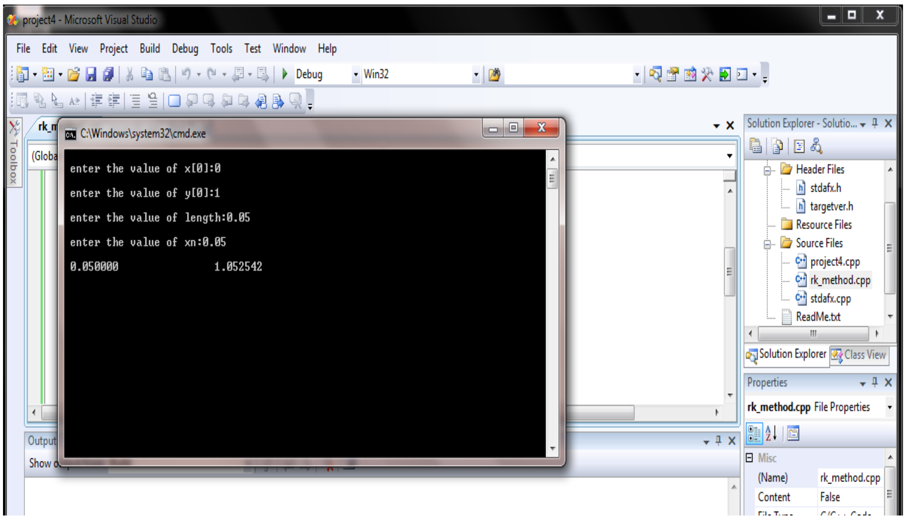

Fig.6 Program results for step size h =0.05

C. ***Discussion of Results:*** *The obtained results are shown in Tables I & II for Numerical methods and* Figures-1, 2, 3, 4 ,5 ,6 for Programming. Graphical representations are based on programming and analytical methods which was show in Figures 7-9.

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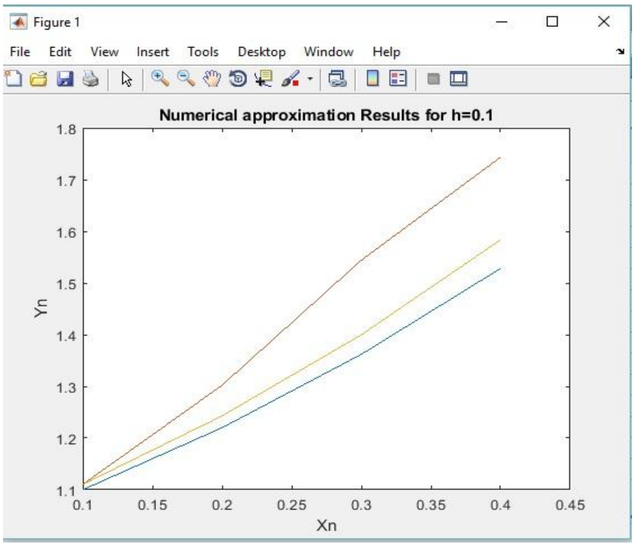

Fig 7 Numerical approximations in Euler (red line), RKmethod (blue line) And Exact values (yellow line) for step size h =0.1

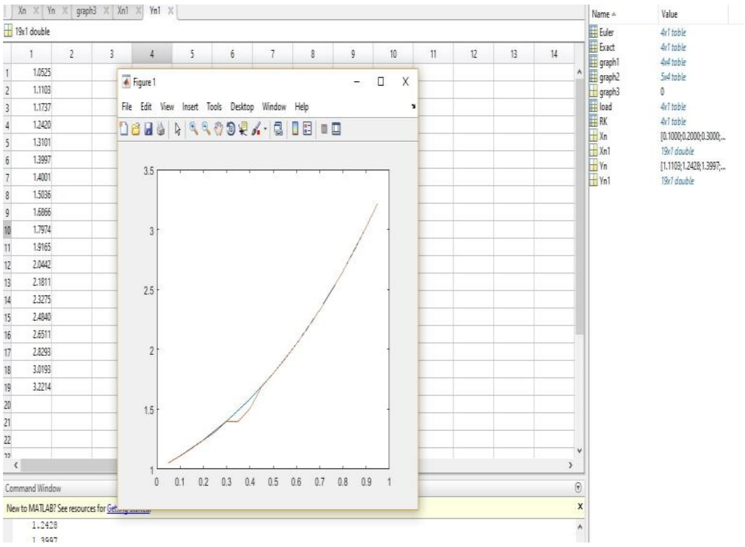

Fig 8 Different step sizes using RKmethod Programming values

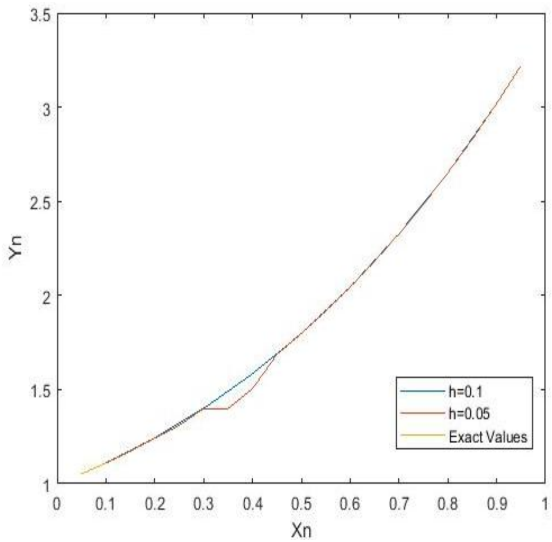

Fig 9 Comparison between RKmethod values and exact values

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Conclusion

In this paper, Euler method and Runge - Kutta method are used for solving ordinary differential equation (ODE) in initial value problems (IVP). Finding more accurate results needs the step size smaller for all methods. Comparing the results of the two methods under investigation, the Euler method was found to be less accurate due to the inaccurate numerical results that were obtained from the approximate solution & programming result in comparison to the exact solution. From the study, the Runge - Kutta method was found to be generally more accurate and also the approximate solution converged faster to the exact solution which was also shown in Figures above. The results obtained through programming for Runge Kutta method is exact, effective and making it easy to bypass analytical complex calculations.

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